

BACHELOR OF SCIENCE (B.Sc.)

Term-End Examination

June, 2015

PHYSICS

PHE-04 : MATHEMATICAL METHODS IN

PHYSICS-I

Time : $1\frac{1}{2}$ hours

Maximum Marks : 25

Note : Attempt *all* questions. The marks for each question are indicated against it. Symbols have their usual meanings. You may use log tables or non-programmable calculators.

1. Attempt any *three* parts :

3×4=12

(a) Vectors \vec{a} and \vec{b} are given by

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}.$$

Calculate (i) the angle between \vec{a} and \vec{b} and (ii) the projection of the vector $\vec{a} + \frac{1}{2}\vec{b}$ onto \vec{a} .

(b) Show that $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0$ for a vector field \vec{F} .

(c) Consider a force $\vec{F} = -x\hat{i} + y\hat{j} + z\hat{k}$ N, acting at a point $P(7\hat{i} + 3\hat{j} + \hat{k})$ m. What is the torque (in Nm) about the origin ?

(d) Calculate the work done in moving a particle in a force field given by

$\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve C defined by $x = t^2 + 1$; $y = 2t^2$; $z = t^3$ from $t = 1$ to $t = 2$.

(e) The nuclear force between two neutrons in a nucleus is described by the Yukawa potential

$$U(r) = -U_0 \frac{r_0}{r} \exp\left(-\frac{r}{r_0}\right)$$

where r is the distance between neutrons and U_0 and r_0 are constants. Determine the force $\vec{F}(r) = -\vec{\nabla} U(r)$.

2. Evaluate $I_{xy} = \iint_R \sigma xy \, dx \, dy$

for a square plate of side L and mass m . σ is mass density of the plate and R is the region of space covered by it.

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OR

State divergence theorem. Evaluate the surface

integral $I = \oiint_S \vec{a} \cdot d\vec{S}$, where

$\vec{a} = (y - x)\hat{i} + y\hat{j} + (z + x^2)\hat{k}$ and S is the closed surface of the sphere $x^2 + y^2 + z^2 = a^2$. $1+4=5$

3. There are 20 computers in a store. Among them, 15 are new and 5 are refurbished, but these are indistinguishable. Six computers are selected at random from this store and purchased for a lab. Compute the probability that among the chosen computers, two are refurbished. 3

OR

A biased dice has probabilities $P(x_i) = \frac{p}{2}, p, p, p, p, 2p$ of showing $x_i = 1, 2, 3, 4, 5, 6$, respectively. Calculate $E(X)$ and $E(X^2)$ for this distribution. 3

4. Derive the expressions for the mean and variance of the normal distribution with mean μ and variance σ^2 given by

$$n(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma} \exp\left[-\frac{1}{2}\left(\frac{x - \mu}{\sigma}\right)^2\right]$$

$$-\infty < x < \infty. \quad 5$$

OR

The heat capacity of liquid sulphuric acid was measured at various temperatures yielding the following set of data :

Heat capacity (in cal C ⁻¹)	Temperature (in °C)
0.38	50
0.39	100
0.40	150
0.41	200
0.45	250
0.46	300

Compute the correlation coefficient r_{xy} .

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