BACHELOR OF SCIENCE (B.Sc.)

Term-End Examination

December, 2015

PHYSICS

PHE-04 : MATHEMATICAL METHODS IN PHYSICS-I

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

Note : *Attempt **all** questions. The marks for each question are indicated against it. Symbols have their usual meanings. You may use log tables or non-programmable calculators.*

1. Attempt any ***three*** parts : *3×4=12*

(a) Determine the volume of a parallelepiped, whose three sides are given to be

$\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

(b) Show that $\vec{\nabla} \cdot (\phi \vec{A}) = (\vec{\nabla}\phi) \cdot \vec{A} + \phi(\vec{\nabla} \cdot \vec{A})$ for a scalar field ϕ and a vector field $\vec{A}$.

(c) Determine the values of the constants a, b and c such that the vector field

$$\vec{A} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$

is irrotational.

(d) If $\vec{A} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, evaluate $\int_C \vec{A} \cdot d\vec{r}$ along the straight line paths from (0, 0, 0) to (1, 0, 0), then from (1, 0, 0) to (1, 1, 0), and then from (1, 1, 0) to (1, 1, 1).

(e) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\vec{\nabla} r^n = nr^{n-2}\vec{r}$.

2. State divergence theorem. Using it, evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. 5

OR

Using Green's theorem, evaluate

$\oint_C [(3x + 4y)dx + (2x - 3y)dy]$ where C is a circle of radius 2 with its centre at the origin of the xy plane. 5

3. There are two groups : a group of 5 boys and another of 10 girls. First, one of the groups – the boys or the girls – is selected. Then one child from among that group is selected. What is the probability that a girl will be selected ? 3

OR

A random variable X has the following density function $f(x) = \begin{cases} \alpha e^{-\alpha x} & , \quad x \geq 0 \\ 0 & , \quad x < 0 \end{cases}$

Determine the variance of this distribution, given that the mean is $\frac{1}{\alpha}$. 3

4. A mass M is suspended from the centre of a steel bar supported at its ends. The depression y is measured by means of a dial height indicator. The following readings are obtained :

M (kg)	0	1	2	3	4
y (μm)	1600	1300	950	600	250

Calculate the best value of the slope and the standard error of estimate for y. 5

OR

Two types of e-mails arrive independently and at random : external e-mails at a mean rate of one every five minutes and internal e-mails at a mean rate of two every five minutes. Calculate the probability of receiving two or more e-mails in any two-minute interval. 5